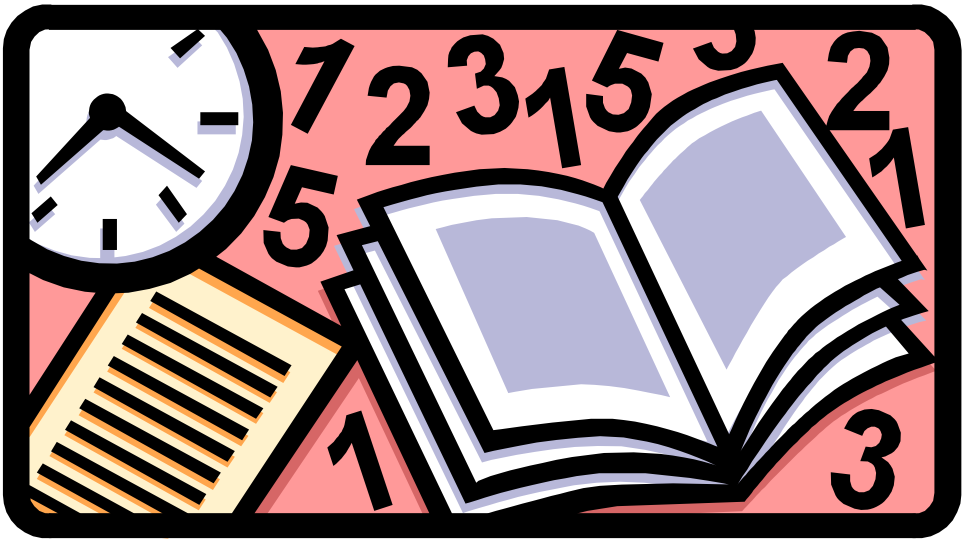
St Augustine's High School



Numeracy Booklet

A guide for pupils,
parents and staff.

Acknowledgment to James Gillespie's High School

Introduction

What is the purpose of the booklet?

This booklet has been produced to give guidance to pupils and parents on how common Numeracy topics are taught in Mathematics and throughout the school. The booklet is available on the school website and in the resources folder for staff. Staff from all departments were consulted during its production. It is hoped that using a consistent approach across all subjects will make it easier for pupils to progress.

How can it be used?

Parents

If you are helping your child with their homework, you can refer to the booklet to see what methods are being taught in school. The booklet includes the Numeracy skills useful in subjects across the curriculum. Look up the relevant page for a step by step guide. You can use the booklet at home to help your child solve number and information handling questions in any subject.

Pupils

The booklet includes the Numeracy skills useful in subjects across the curriculum. You should keep it with you in school at all times. You can refer to it for help with number and information handling questions in homework and classwork.

Look up the relevant page for a step by step guide.

For help with mathematics topics, you should refer to your mathematics textbook or ask your teacher for help.

Why do some topics include more than one method?

In some cases (e.g. percentages), the method used will be dependent on the level of difficulty of the question, and whether or not a calculator is permitted.

For mental calculations, pupils should be encouraged to develop a variety of strategies so that they can select the most appropriate method in any given situation.

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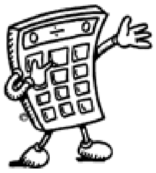
Addition

Mental strategies



There are a number of useful mental strategies for addition. Some examples are given below.

Example Calculate 54 + 27

Method 1 Add tens, then add units, then add together

50 + 20 = 70 4 + 7 = 11 70 + 11 = 81

Method 2 Split up number to be added into tens and units and add separately.

54 + 20 = 74 74 + 7 = 81

Method 3 Round up to nearest 10, then subtract

54 + 30 = 84 but 30 is 3 too much so subtract 3;
84 - 3 = 81

Written Method

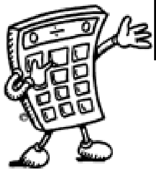
When adding numbers, ensure that the numbers are lined up according to place value. Start at right hand side, write down units, carry tens.

Example Add 3032 and 589

```
3032         3032          3032          3032
+589    →    +589    →     +589    →     +589
   1           21           621          3621
```

2 + 9 = 11 3+8+1=12 0+5+1=6 3 + 0 = 3

Subtraction



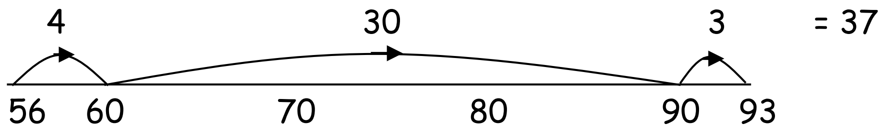
We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

Mental Strategies

Example Calculate 93 - 56

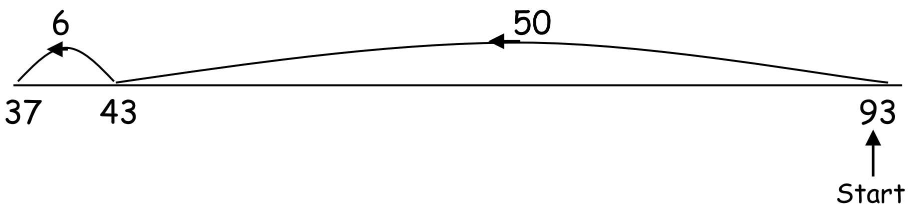
Method 1 Count on

Count on from 56 until you reach 93. This can be done in several ways e.g.



Method 2 Break up the number being subtracted

e.g. subtract 50, then subtract 6 93 - 50 = 43
43 - 6 = 37



Written Method

Example 1 4590 – 386

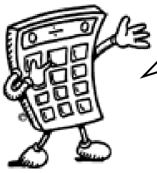
```
   8 1
  4590
 - 386
  4204
```

We do not "borrow and pay back".

Example 2 Subtract 692 from 14597

```
 3 1
 14597
 -  692
 13905
```

Multiplication 1



It is essential that you know all of the multiplication tables from 1 to 10. These are shown in the tables square below.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Mental Strategies

Example Find 39 x 6

Method 1

30 x 6
= 180

9 x 6
= 54

180 + 54
= 234

Method 2

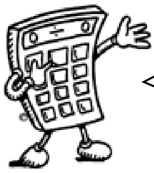
40 x 6
=240

40 is 1 too many
so take away 6x1

240 - 6
= 234

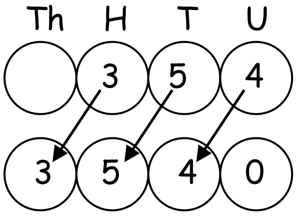
Multiplication 2

Multiplying by multiples of 10 and 100

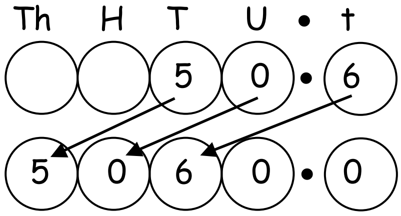


To multiply by **10** you move every digit ***one*** place to the left.
To multiply by **100** you move every digit ***two*** places to the left.

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100



354 x 10 = 3540



50.6 x 100 = 5060

(c) 35 x 30

To multiply by 30, multiply by 3, then by 10.

35 x 3 = 105
105 x 10 = 1050

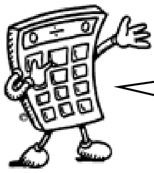
so 35 x 30 = 1050

(d) 436 x 600

To multiply by 600, multiply by 6, then by 100.

436 x 6 = 2616
2616 x 100 = 261600

so 436 x 600 = 261600



We may also use these rules for multiplying decimal numbers.

Example 2 (a) 2.36 x 20

2.36 x 2 = 4.72
4.72 x 10 = 47.2

so 2.36 x 20 = 47.2

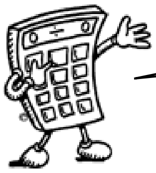
(b) 38.4 x 50

38.4 x 5 = 192.0
192.0x 10 = 1920

so 38.4 x 50 = 1920

Division

You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.



Written Method

Example 1 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r} 2\;4 \\ 8\,\overline{)\,1\;9^{3}2} \end{array}$$

There are 24 pupils in each class

Example 2 Divide 4.74 by 3

$$\begin{array}{r} 1\;.\;5\;8 \\ 3\,\overline{)\,4\;.\;^{1}7^{2}4} \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 3 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$\begin{array}{r} 0\;.\;2\;7\;5 \\ 8\,\overline{)\,2\;.\;^{2}2^{6}0^{4}0} \end{array}$$

Each glass contains 0.275 litres

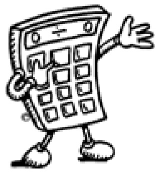
If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

Order of Calculation (BOMDAS)

Consider this: What is the answer to 2 + 5 x 8 ?

Is it 7 x 8 = 56 or 2 + 40 = 42 ?

The correct answer is 42.



Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BOMDAS**

The **BOMDAS** rule tells us which operations should be done first.
BOMDAS represents:

(B)rackets
(O)f
(M)ultiply
(D)ivide
(A)dd
(S)ubract

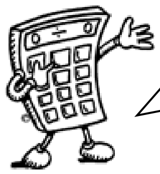
Scientific calculators use this rule, some basic calculators may not, so take care in their use.

Example 1 15 – 12 ÷ 6 BOMDAS tells us to divide first
= 15 – 2
= 13

Example 2 (9 + 5) x 6 BOMDAS tells us to work out the brackets first
= 14 x 6
= 84

Example 3 18 + 6 ÷ (5-2) Brackets first
= 18 + 6 ÷ 3 Then divide
= 18 + 2 Now add
= 20

Evaluating Formulae



To find the value of a variable in a formula, we must substitute all of the given values into the formula, then use BODMAS rules to work out the answer.

Example 1

Use the formula $P = 2L + 2B$ to evaluate P when $L = 12$ and $B = 7$.

$P = 2L + 2B$	Step 1: write formula
$P = 2 \times 12 + 2 \times 7$	Step 2: substitute numbers for letters
$P = 24 + 14$	Step 3: start to evaluate (BODMAS)
$P = 38$	Step 4: write answer

Example 2

Use the formula $I = \frac{V}{R}$ to evaluate I when $V = 240$ and $R = 40$

$$I = \frac{V}{R}$$

$$I = \frac{240}{40}$$

$$I = 6$$

Example 3

Use the formula $F = 32 + 1.8C$ to evaluate F when $C = 20$

$F = 32 + 1.8C$

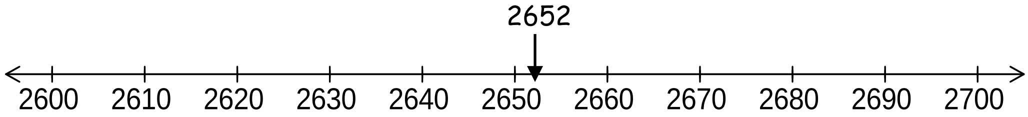
$F = 32 + 1.8 \times 20$

$F = 32 + 36$

$F = 68$

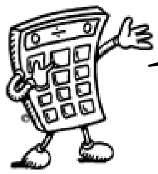
Estimation : Rounding

Numbers can be rounded to give an approximation.



26**5**2 rounded to the nearest 10 is 2650.
2**6**52 rounded to the nearest 100 is 2700.

When rounding numbers which are exactly in the middle, convention is to **round up**.
78**6**5 rounded to the nearest 10 is 78**7**0.



The same principle applies to rounding decimal numbers.

In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right (the "check digit") - if it is 5 or more round up.

Example 1 Round 46 753 to the nearest thousand.

6 is the digit in the thousands column - the check digit (in the hundreds column) is a 7, so round up.

4**6** 753
= 47 000 to the nearest thousand

Example 2 Round 1.57359 to 2 decimal places

The second number after the decimal point is a 7 - the check digit (the third number after the decimal point) is a 3, so round down.

1.5**7**359
= 1.57 to 2 decimal places

Significant Figures

Rules for significant figures:

1. All non zero figures (digits) are significant
2. Zeroes which are blocked in are significant
3. Zeros at the end of a decimal are significant
4. Zeros which are not blocked in are not significant

Examples

743 has 3 significant figures (3sf) (rule 1)

4038 has 4sf (rule 2)

14·60 has 4sf (rule 3)

5200 has 2sf (rule 4)

0·0038 has 2sf (rule 4)

Rounding to a given number of significant figures

This follows the same rules as any rounding

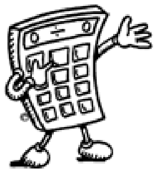
If the check digit is 5 or more you round up (≥ 5 round up)

If the check digit is less than 5 then round down (< 5 round down)

Examples

Rounding 472 to 2sf = 470 since check digit is 2 which is < 5
Rounding 628 to 2sf = 630 since check digit is 8 which is ≥ 5
Rounding 3·74 to 2sf = 3·7 since check digit is 4 which is < 5
Rounding 18·37 to 3sf = 18·4 since check digit is 7 which is ≥ 5
Rounding 5267 to 3sf = 5270 since check digit is 7 which is ≥ 5
Rounding 12539 to 3sf = 12500 since check digit is 3 which is < 5
Rounding 0·427 to 2sf = 0·43 since check digit is 7 which is ≥ 5
Rounding 0·0683 to 2sf = 0·068 since check digit is 3 which is < 5
Rounding 6·081 to 2sf = 6·1 since check digit is 8 which is ≥ 5
Rounding 6·029 to 2sf = 6·0 since check digit is 2 which is < 5

Estimation : Using Rounding



Example 1
Tickets for a concert were sold over 4 days. The number of tickets sold each day was recorded in the table below. How many tickets were sold in total?

Monday	Tuesday	Wednesday	Thursday
486	205	197	321

Estimate = 500 + 200 + 200 + 300 = 1200

Calculate:

```
  486
  205
  197
 +321
 1209
```

Answer = 1209 tickets

Example 2
A bar of chocolate weighs 42g. There are 48 bars of chocolate in a box. What is the total weight of chocolate in the box?

Estimate = 50 x 40 = 2000g

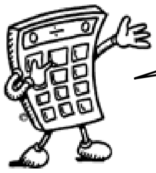
Calculate:

```
   42
  x48
  336
 1680
 2016
```

Answer = 2016g

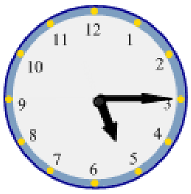
Time 1

Time may be expressed in 12 or 24 hour notation.



12-hour clock

Time can be displayed on a clock face, or digital clock.

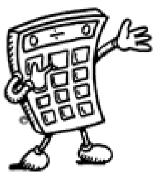


05: 15

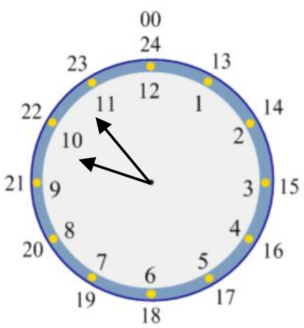
These clocks both show fifteen minutes past five, or quarter past five.

When writing times in 12 hour clock, we need to add a.m. or p.m. after the time.
a.m. is used for times between midnight and 12 noon (morning)
p.m. is used for times between 12 noon and midnight (afternoon / evening).

24-hour clock



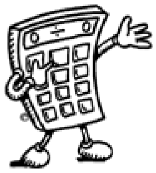
In 24 hour clock, the hours are written as numbers between 00 and 24. Midnight is expressed as 00 00, or 24 00. After 12 noon, the hours are numbered 13, 14, 15 … etc.



Examples

9.55 am ⟶ 09 55 hours
3.35 pm ⟶ 15 35 hours
12.20 am ⟶ 00 20 hours
02 16 hours ⟶ 2.16 am
20 45 hours ⟶ 8.45 pm

Time 2



It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time Facts

In 1 year, there are: 365 days (366 in a leap year)
52 weeks
12 months

The number of days in each month can be remembered using the rhyme:

"30 days hath September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year."

Distance, Speed and Time.

For any given journey, the distance travelled depends on the speed and the time taken. If speed is constant, then the following formulae apply:

$$\text{Distance} = \text{Speed} \times \text{Time} \quad \text{or} \quad D = S\,T$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad S = \frac{D}{T}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \quad \text{or} \quad T = \frac{D}{S}$$

Example Calculate the speed of a train which travelled 450 km in 5 hours

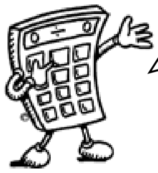
$$S = \frac{D}{T}$$

$$S = \frac{450}{5}$$

$$S = 90 \text{ km/h}$$

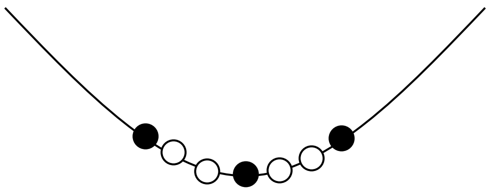
Fractions 1

Addition, subtraction, multiplication and division of fractions are studied in mathematics.
However, the examples below may be helpful in all subjects.



Understanding Fractions

Example
A necklace is made from black and white beads.



What fraction of the beads are black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ of the beads are black.

Equivalent Fractions

Example
What fraction of the flag is shaded?

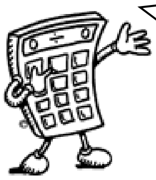
6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Fractions 2

Simplifying Fractions



The top of a fraction is called the **numerator**, the bottom is called the **denominator**.
To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the same number.

Example 1

(a) ÷5

$$\frac{20}{25} = \frac{4}{5}$$

÷5

(b) ÷8

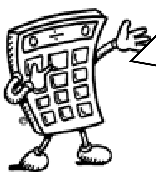
$$\frac{16}{24} = \frac{2}{3}$$

÷8

This can be done repeatedly until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in it's **simplest form**.

Example 2 Simplify $\frac{72}{84}$ $\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7}$ (simplest form)

Calculating Fractions of a Quantity



To find the fraction of a quantity, divide by the denominator.
To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7 etc.

Example 1 Find $\frac{1}{5}$ of £150

$\frac{1}{5}$ of £150 = £150 ÷ 5 = £30

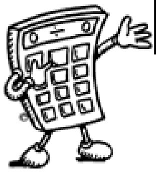
Example 2 Find $\frac{3}{4}$ of 48

$\frac{1}{4}$ of 48 = 48 ÷ 4 = 12

so $\frac{3}{4}$ of 48 = 3 x 12 = 36

To find $\frac{3}{4}$ of a quantity, start by finding $\frac{1}{4}$

Percentages 1



Percent means out of 100.
A percentage can be converted to an equivalent fraction or decimal.

36% means $\frac{36}{100}$

36% is therefore equivalent to $\frac{9}{25}$ and 0.36

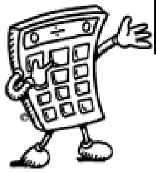
Common Percentages

Some percentages are used very frequently. It is useful to know these as fractions and decimals.

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	0.01
10%	$\frac{1}{10}$	0.1
20%	$\frac{1}{5}$	0.2
25%	$\frac{1}{4}$	0.25
$33\frac{1}{3}$%	$\frac{1}{3}$	0.333…
50%	$\frac{1}{2}$	0.5
$66\frac{2}{3}$%	$\frac{2}{3}$	0.666…
75%	$\frac{3}{4}$	0.75

Percentages 2

There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.



Non- Calculator Methods

Method 1 Using Equivalent Fractions

Example Find 25% of £640

25% of £640 $= \frac{1}{4}$ of £640 = £640 ÷ 4 = £160

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example Find 9% of 200g

1% of 200g = $\frac{1}{100}$ of 200g = 200g ÷ 100 = 2g

so 9% of 200g = 9 x 2g = 18g

Method 3 Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example Find 70% of £35

10% of £35 = $\frac{1}{10}$ of £35 = £35 ÷ 10 = £3.50

so 70% of £35 = 7 x £3.50 = £24.50

Percentages 3

Non- Calculator Methods (continued)

The previous 2 methods can be combined so as to calculate any percentage.

Example Find 23% of £15000

10% of £15000 = £1500 so 20% = £1500 x 2 = £3000
1% of £15000 = £150 so 3% = £150 x 3 = £450

23% of £15000 = £3000 + £450 = £3450

Finding $17\frac{1}{2}$% (without a calculator)

firstly find 10%

Example Calculate the total price of a computer which costs £650 excluding a delivery charge of $17\frac{1}{2}$%

10% of £650 = £65 (divide by 10)
5% of £650 = £32.50 (divide previous answer by 2)
$2\frac{1}{2}$% of £650 = £16.25 (divide previous answer by 2)

so $17\frac{1}{2}$% of £650 = £65 + £32.50 + £16.25 = £113.75

Total price = £650 + £113.75 = £763.75

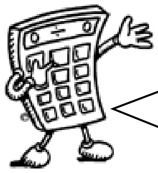
Percentages 4

Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Example 1 Find 23% of £15000

23% = 0.23 so 23% of £15000 = 0.23 x £15000 = £3450



We do not use the % button on calculators. The methods taught in the mathematics department are all based on converting percentages to decimals.

Example 2 House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236000 at the start of the year?

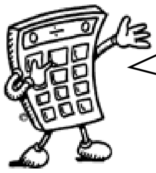
19% = 0.19 so Increase = 0.19 x £236000
= £44840

Value at end of year = original value + increase
= £236000 + £44840
= £280840

The new value of the house is £280840

Percentages 5

Finding the percentage



To find a percentage of a total, first make a fraction, then convert to a decimal by dividing the top by the bottom. This can then be expressed as a percentage.

Example 1 There are 30 pupils in Class 3A3. 18 are girls. What percentage of Class 3A3 are girls?

$\frac{18}{30} = 18 \div 30 = 0.6 = 60\%$

60% of 3A3 are girls

Example 2 James scored 36 out of 44 his biology test. What is his percentage mark?

Score = $\frac{36}{44} = 36 \div 44 = 0.81818\ldots$

$= 81.818..\% = 82\%$ (rounded)

Example 3 In class 1X1, 14 pupils had brown hair, 6 pupils had blonde hair, 3 had black hair and 2 had red hair. What percentage of the pupils were blonde?

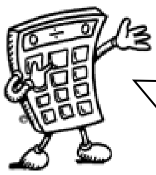
Total number of pupils = 14 + 6 + 3 + 2 = 25

6 out of 25 were blonde, so,

$\frac{6}{25} = 6 \div 25 = 0.24 = 24\%$

24% were blonde.

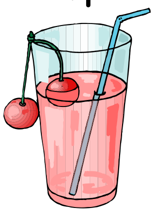
Ratio 1



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.
The ratio of water to cordial is 4:1
(said "4 to 1")
The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2

In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is 5 : 7 : 8

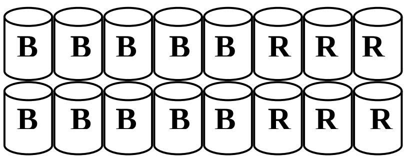
Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



Blue : Red = 10 : 6
= 5 : 3

To simplify a ratio, divide each figure in the ratio by a common factor.

Ratio 2

Simplifying Ratios (continued)

Example 2
Simplify each ratio:
(a) 4:6 (b) 24:36 (c) 6:3:12

(a) 4:6
= 2:3
Divide each figure by 2

(b) 24:36
= 2:3
Divide each figure by 12

(c) 6:3:12
= 2:1:4
Divide each figure by 3

Example 3
Concrete is made by mixing 20 kg of sand with 4 kg cement. Write the ratio of sand : cement in its simplest form

Sand : Cement = 20 : 4
= 5 : 1

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

Fruit	Nuts
3	2
15	**10**

(×5 on each side)

So the chocolate bar will contain 10g of nuts.

Ratio 3

Sharing in a given ratio

Example

Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3:2. How much money did each receive?

Step 1 Add up the numbers to find the total number of parts

$3 + 2 = 5$

Step 2 Divide the total by this number to find the value of each part

$90 \div 5 =$ £18

Step 3 Multiply each figure by the value of each part

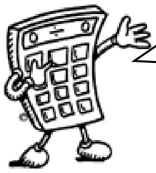
3 x £18 = £54
2 x £18 = £36

Step 4 Check that the total is correct

£54 + £36 = £90 ✓

Lauren received £54 and Sean received £36

Proportion



Two quantities are said to be in direct proportion if when one doubles the other doubles.
We can use proportion to solve problems.

It is often useful to make a table when solving problems involving proportion.

Example 1
A car factory produces 1500 cars in 30 days. How many cars would they produce in 90 days?

Days	Cars
30	1500
90	**4500**

(×3 on both sides)

The factory would produce 4500 cars in 90 days.

Example 2
5 adult tickets for the cinema cost £27.50. How much would 8 tickets cost?

Find the cost of 1 ticket →

Tickets	Cost
5	£27.50
1	£5.50
8	£44.00

Working:

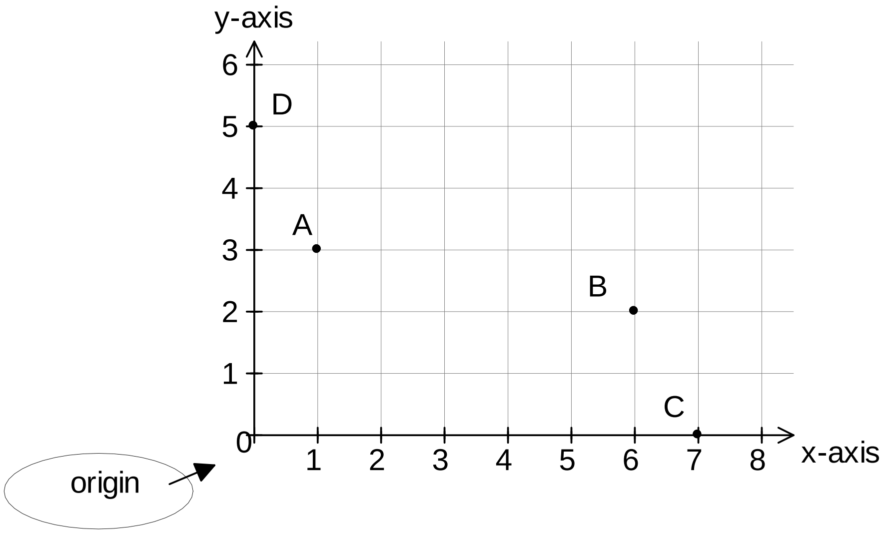
$5 \overline{)\,£27.50}$ = £5.50

£5.50 × 8 = £44.00

The cost of 8 tickets is £44

Coordinates

Coordinates are used to give positions and points on a grid



For coordinates you go along first, then up.
Coordinates are written as 2 numbers in brackets separated by a comma. So:

"A is at position (1, 3)" or has "A has coordinates (1, 3)"

"B is at position (6, 2)" or has "B has coordinates (6, 2)"

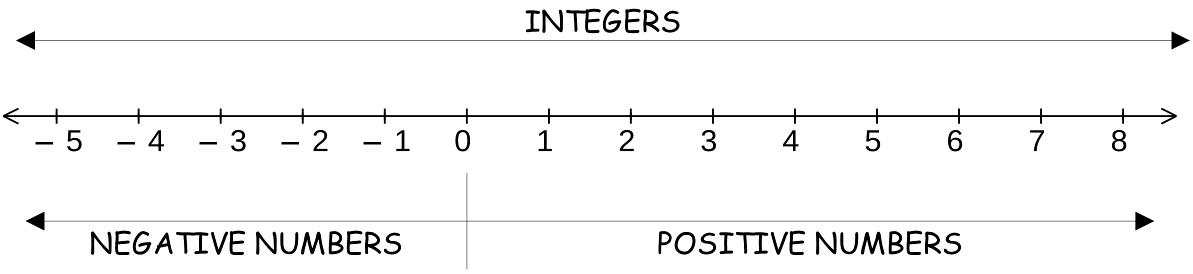
"C is at position (7, 0)" or has "C has coordinates (7, 0)"

"D is at position (0, 5)" or has "D has coordinates (0, 5)"

The horizontal line is called the x-axis.
The vertical line is called the y-axis.

Adding and Subtracting Integers

Numbers below zero are called NEGATIVE NUMBERS
Applications for negative numbers include temperatures and coordinates
When you look at negative and positive whole numbers including zero you are dealing with INTEGERS.
Number lines can be used to help with integer problems. Number lines can be drawn horizontally or vertically. Vertical number lines can be referenced to thermometers.



<u>Adding</u>
Adding takes you right on the number line

$^-5 + 3 = ^-2$ $^-3 + 7 = 4$ $^-6 + 6 = 0$

<u>Subtracting</u>
Subtracting takes you left on the number line

$3 - 5 = ^-2$ $4 - 8 = ^-4$ $^-2 - 3 = ^-5$

<u>More complicated Adding and Subtracting</u>
Look at these patterns

$3 + 2 = 5$	$5 - 2 = 3$
$3 + 1 = 4$	$5 - 1 = 4$
$3 + 0 = 3$	$5 - 0 = 5$
$3 + ^-1 = 2$	$5 - ^-1 = 6$
$3 + ^-2 = 1$	$5 - ^-2 = 7$
$3 + ^-3 = 0$	$5 - ^-3 = 8$
$3 + ^-4 = ^-1$	$5 - ^-4 = 9$
$3 + ^-5 = ^-2$	$5 - ^-5 = 10$
Adding a negative is the same as subtracting	Subtracting a negative is the same as adding
$2 + ^-6$	$4 - ^-5$
$= 2 - 6$	$= 4 + 5$
$= ^-4$	$= 9$

Multiplying and Dividing Integers

Multiplying Integers

Look at these patterns

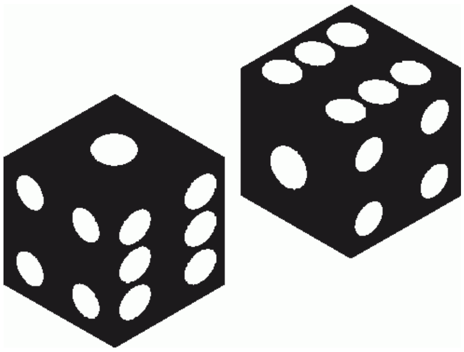
$3 \times 3 = 9$	$^-2 \times 3 = {}^-6$
$3 \times 2 = 6$	$^-2 \times 2 = {}^-4$
$3 \times 1 = 3$	$^-2 \times 1 = {}^-2$
$3 \times 0 = 0$	$^-2 \times 0 = 0$
$3 \times {}^-1 = {}^-3$	$^-2 \times {}^-1 = 2$
$3 \times {}^-2 = {}^-6$	$^-2 \times {}^-2 = 4$
$3 \times {}^-3 = {}^-9$	$^-2 \times {}^-3 = 6$
$3 \times {}^-4 = {}^-12$	$^-2 \times {}^-4 = 8$
If one value is negative the answer is negative	If both values are negative the answer is positive
$4 \times {}^-5 = {}^-20$	$^-5 \times {}^-4 = 20$

Dividing

The above rules also hold true for dividing

$20 \div -4 = {}^-5$ $\quad$ $^-18 \div 6 = {}^-3$ $\quad$ $^-28 \div -4 = 7$

Probability



Probability deals with putting a value on the chances of something happening
Probability is expressed as a value between 0 and 1.
It is often expressed as a fraction.

Example 1
Probability of getting a 2 when you roll a die

is $\dfrac{\text{1 chance: only one 2 on a die}}{\text{6 chances: 6 different possible numbers}} = \dfrac{1}{6}$

Example 2
Probability of picking an even number at random from this list
2, 3, 5, 7, 8, 15, 17

is $\dfrac{\text{2 chances: two even numbers}}{\text{7 choices: 7 different possible numbers}} = \dfrac{2}{7}$

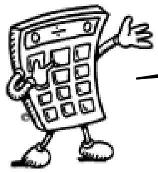
Example 3

Probability of picking a blue pencil from a box of 5 red, 2 blue and 3 green pencils is

$\dfrac{\text{2 chances: two blue}}{\text{10 choices: 10 pencils to choose from}} = \dfrac{2}{10} = \dfrac{1}{5}$ when simplified

Information Handling : Tables

It is sometimes useful to display information in graphs, charts or tables.



Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	**24**	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is 24°C

Frequency Tables are used to present information. Often data is grouped in intervals.

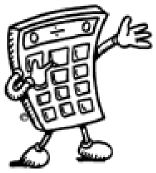
Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

Mark	Tally	Frequency
16 - 20	||	2
21 - 25	~~||||~~ ||	7
26 - 30	~~||||~~ ||||	9
31 - 35	~~||||~~	5
36 - 40	|||	3
41 - 45	||	2
46 - 50	||	2

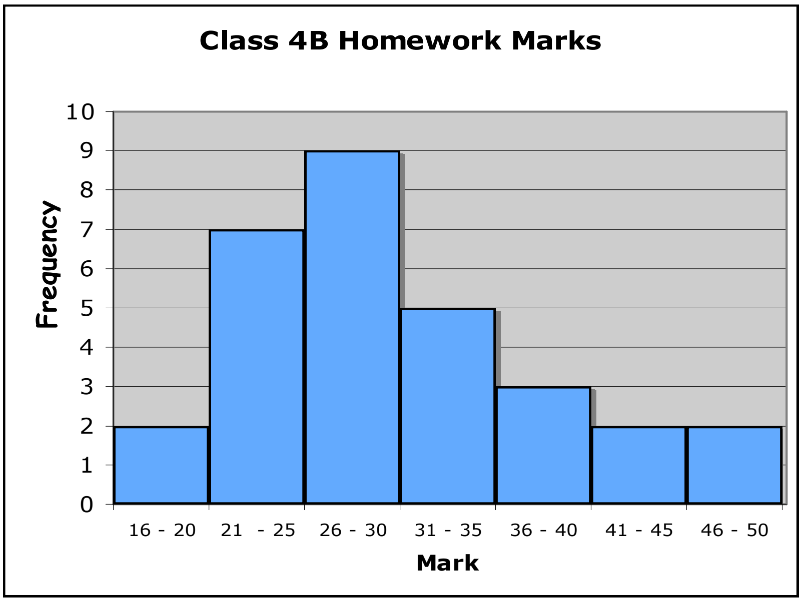
Each mark is recorded in the table by a tally mark.
Tally marks are grouped in 5's to make them easier to read and count.

Information Handling : Bar Graphs

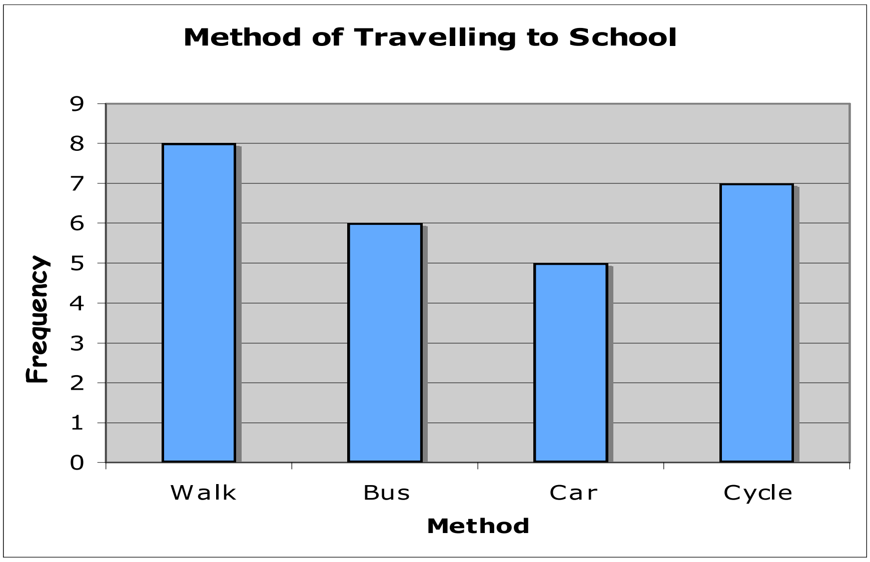


Bar graphs are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency. All graphs should have a title, and each axis must be labelled.

Example 1 The graph below shows the homework marks for Class 4B.

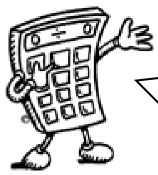


Example 2 How do pupils travel to school?



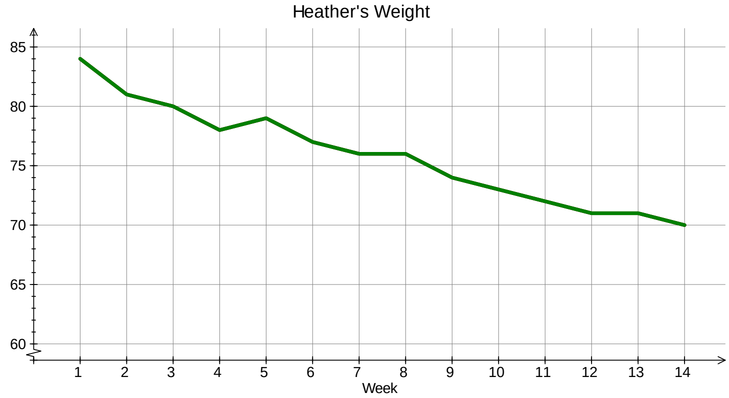
When the horizontal axis shows categories, rather than grouped intervals, it is common practice to leave gaps between the bars.

Information Handling : Line Graphs



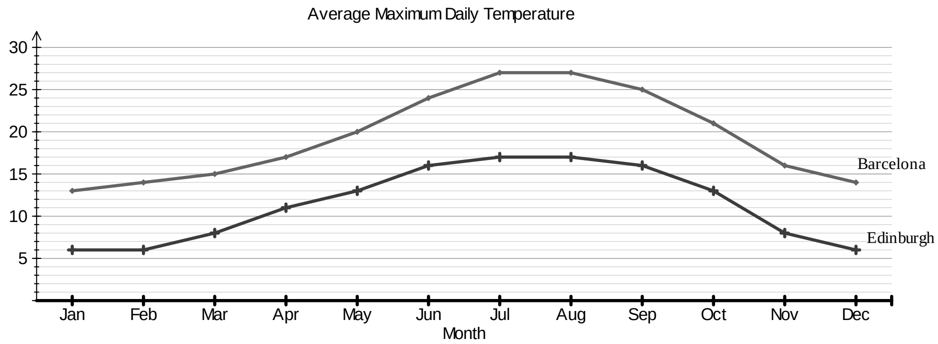
Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and each axis must be labelled including units. The trend of a graph is a general description of it.

Example 1 The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.

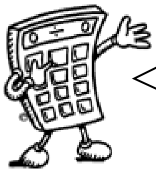


The trend of the graph is that her weight is decreasing.

Example 2 Graph of temperatures in Edinburgh and Barcelona.



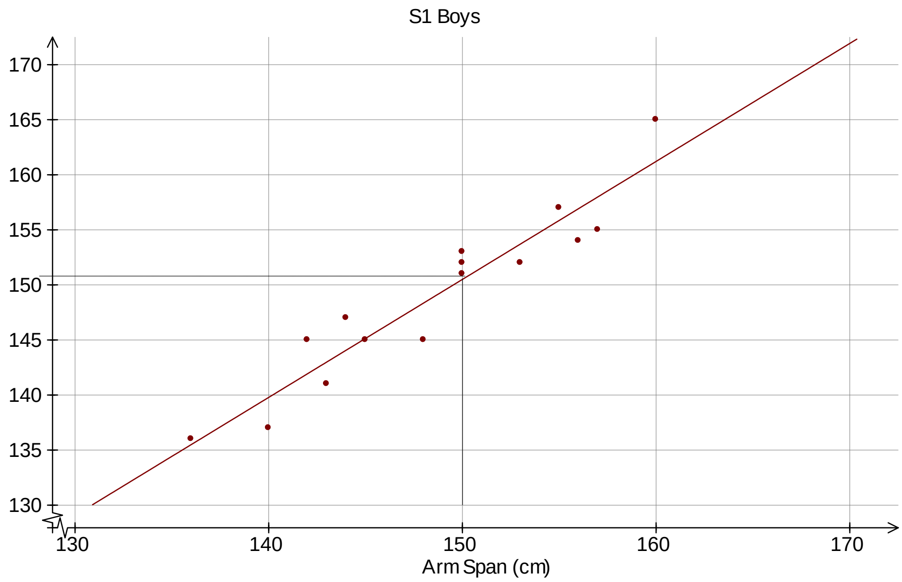
Information Handling : Scatter Graphs



A scatter diagram is used to display the relationship between two variables.
A pattern may appear on the graph. This is called a **correlation**.

Example The table below shows the height and arm span of a group of first year boys. This is then plotted as a series of points on the graph below.

Arm Span (cm)	150	157	155	142	153	143	140	145	144	150	148	160	150	156	136
Height (cm)	153	155	157	145	152	141	138	145	148	151	145	165	152	154	137

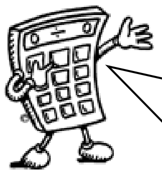


The graph shows a general trend, that as the arm span increases, so does the height. This graph shows a positive correlation.

The line drawn is called the line of best fit. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 151cm.

Note that in some subjects, it is a requirement that the axes start from zero.

Information Handling : Pie Charts



A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example 30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.

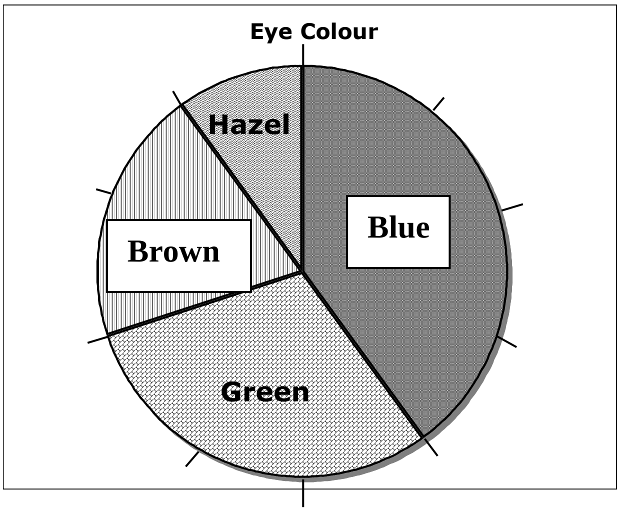


Figure 1

How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

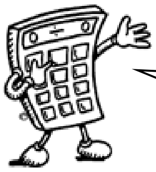
$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

The angle in the brown sector is 72°.
so the number of pupils with brown eyes
= $\frac{72}{360}$ x 30 = 6 pupils.
If finding all of the values, you can check your answers -

Information Handling : Pie Charts 2

Drawing Pie Charts



On a pie chart, the size of the angle for each sector is calculated as a fraction of 360°.

Example: In a survey about television programmes, a group of people were asked what was their favourite soap. Their answers are given in the table below. Draw a pie chart to illustrate the information.

Soap	**Number of people**
Eastenders	28
Coronation Street	24
Emmerdale	10
Hollyoaks	12
None	6

Total number of people = 80

Eastenders $= \frac{28}{80} \rightarrow \frac{28}{80} \times 360° = 126°$

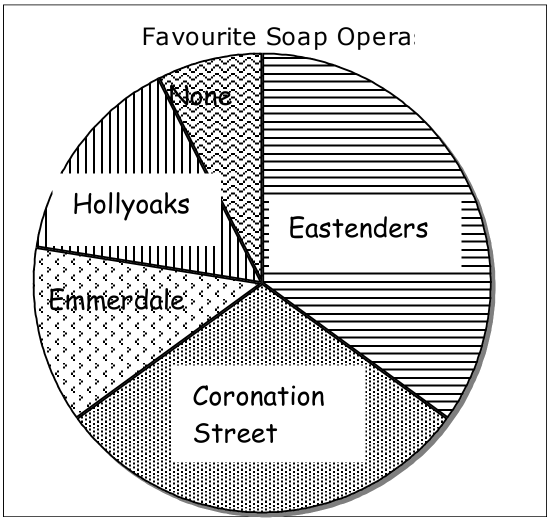
Coronation Street $= \frac{24}{80} \rightarrow \frac{24}{80} \times 360° = 108°$

Emmerdale $= \frac{10}{80} \rightarrow \frac{10}{80} \times 360° = 45°$

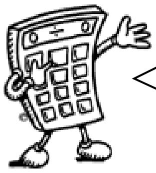
Hollyoaks $= \frac{12}{80} \rightarrow \frac{12}{80} \times 360° = 54°$

None $= \frac{6}{80} \rightarrow \frac{6}{80} \times 360° = 27°$

Check that the total = 360°



Information Handling : Averages



To provide information about a set of data, the average value may be given. There are 3 ways of finding the average value – the mean, the median and the mode.

Mean

The mean is found by adding all the data together and dividing by the number of values.

Median

The median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

Mode

The mode is the value that occurs most often.

Range

The range of a set of data is a measure of spread.

Range = Highest value – Lowest value

Example Class 1A4 scored the following marks for their homework assignment. Find the mean, median, mode and range of the results.

7, 9, 7, 5, 7, 7, 10, 9, 8, 4, 8, 5, 8, 10

Mean = $\frac{7+9+7+5+7+7+10+9+8+4+8+5+8+10}{14}$

= $\frac{104}{14} = 7.4285...$ Mean = 7.4 to 1 decimal place

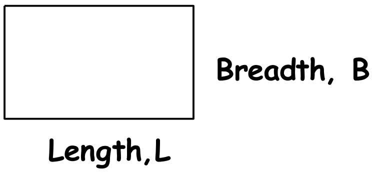
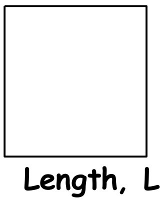
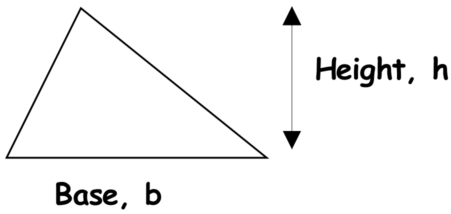
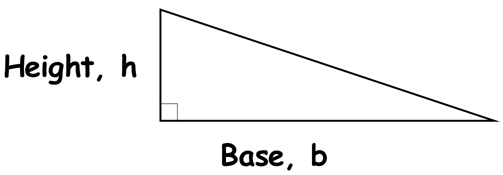
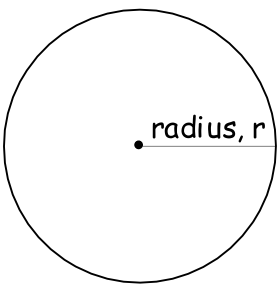
Ordered values: 4, 5, 5, 7, 7, 7, [7, 8,] 8, 8, 9, 9, 10, 10

Median = 7·5

7 is the most frequent mark, so Mode = 7

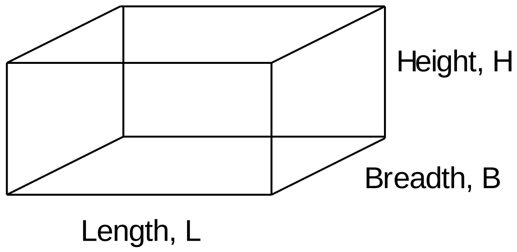
Range = 10 – 4 = 6

Areas of Common 2D Shapes

<u>Rectangle</u>		
	Area $A = L \times B$	or $A = LB$
<u>Square</u>		
	Area $A = L \times L$	or $A = L^2$
<u>Triangles</u>		
 	Area $A = b \times h \div 2$	or $A = \frac{1}{2}bh$
<u>Circle</u>		
	Area $A = \pi r^2$	There is a π button on scientific calculators. Use $\pi = 3{\cdot}14$ for non-calculator questions

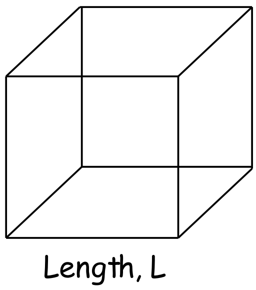
Volume of Common 3D Solids

Cuboid



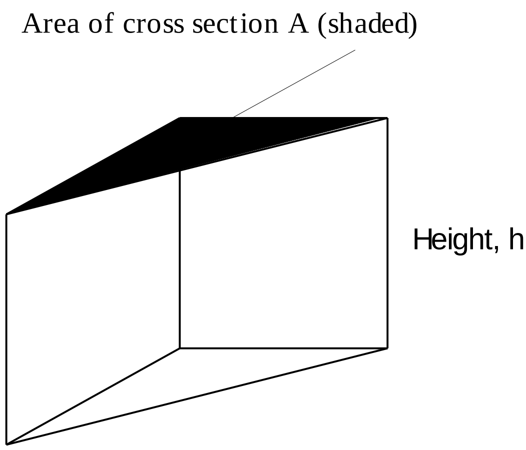
Volume $V = L \times B \times H$ or $V = LBH$

Cube



Volume $V = L \times L \times L$ or $V = L^3$

Prism



Volume $V = A \times h$ or $V = Ah$

Units of Measure (Metric System)

Length
In Mathematics, we use millimetres (mm), centimetres (cm), metres (m) and kilometres (km).

1 mm is about the thickness of a pencil point
1 cm is about the thickness of your smaller finger
1 m is about the height above the ground of a door handle
1 km is about $2\frac{1}{2}$ times round a football pitch.

Unit Conversion		
10mm = 1cm	mm to cm ÷ 10 cm to mm x 10	20mm = 2cm 4cm = 40mm
100cm = 1m	cm to m ÷ 100 m to cm x 100	300cm = 3m 2m = 200cm
1000m = 1km	m to km ÷ 1000 km to m x 1000	5000m = 5km 6km = 6000m

Weight
In Mathematics, we use grammes (g) and kilogrammes (kg) and occasionally tonnes.

Unit Conversion		
1000g = 1kg	g to kg ÷ 1000 kg to g x 1000	3000g = 3kg 2kg = 2000g
1000kg = 1tonne	kg to tonne ÷ 1000 tonne to kg x 1000	5000kg = 5tonnes 6tonnes = 6000kg

Capacity
In Mathematics we use millilitres (ml) and litres. We also use the facts that 1ml = $1cm^3$ and that 1ml of water weighs 1g (at 4 °C) so 1 litre of water weighs 1 kg.

Unit Conversion		
1000ml = 1litre	ml to litre ÷ 1000 litre to ml x 1000	5000ml = 5litres 7litres = 7000ml

Mathematical Dictionary (Key words):

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: 12+76 = 88
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 50 – 36 = 14
Division (÷)	Sharing a number into equal parts. 24 ÷ 6 = 4
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. Example: 10 is greater than 6. 10 > 6
Least	The lowest number in a group (minimum).

Less than (<)	Is smaller or lower than. Example: 15 is less than 21. 15 < 21.
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see p38)
Median	Another type of average - the middle number of an ordered set of data (see p38)
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average – the most frequent number or category (see p38)
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example 6 x 4 = 24
Negative Number	A number less than zero. Shown by a minus sign. Example -5 is a negative number.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 ,3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BODMAS (see p9)
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).